$$\rho=\frac{\left[1+\left(\frac{dy_1}{dx_1}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y_1}{dx_1^2}}.$$

The equation of a circle whose center is the point of intersection and which passes through the foot of the normal is

$$(x-x_1)^2+(y-y_1)^2=\rho^2.$$

Differentiating this twice and eliminating x, y, we find

$$\rho=\frac{\left[1+\left(\frac{dy_1}{dx_1}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y_1}{dx_1^2}}.$$

This shows that a circle having the same slope and same value of $\frac{d^2y}{dx^2}$ at its point of intersection with a given curve has its center at the limiting point of intersection of two normals.

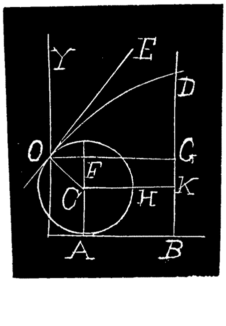
MECHANICS.

100. Proposed by **WALTER H. DRANE, Graduate Student, Harvard University; Cambridge, Mass.**

A man, riding a bicycle, runs through a puddle of water and a bit of mud is thrown from the rear wheel and alights on the crown of his hat. Supposing the wheel 28 inches in diameter, that the man's head is 6 feet above ground, that the saddle is 1 foot in front of the rear wheel, and that the mud left the wheel at a point 30° from highest point of wheel, how long will it take a man to ride a mile at this rate?

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and L. R. INGERSOLL, Student Colorado College, Colorado Springs, Col.

As the particle of mud and man have the same uniform velocity forward, it is not necessary to consider such motion. The result will be the same if we regard the man at rest and the hind wheel of the bicycle revolving with the same velocity as the man is moving forward. Let O be the origin of the coördinates, D the top of the man's head, $\angle EOG=\angle OCF=\theta$. OE the tangent to the wheel at O, $CO=a=14$ inches, $HK=12$ inches, $BD=72$ inches, $g=32.16$ feet. Then $y=x\tan\theta-gx^2/2v^2\cos^2\theta$, is the equation to OD, the path of the mud.



$$\therefore\ v=\frac{x}{2\cos\theta}\sqrt{\frac{2g}{x\tan\theta-y}}\ldots.(1).$$

$x=OG=OF+FG=OF+CH+HK=26+14\sin\theta$.

$y=GD=BD-BK-KG=72-14-14\cos\theta=58-14\cos\theta$.

$$\therefore\ v=\frac{(13+7\sin\theta)}{\cos\theta}\sqrt{\frac{g}{(13+7\sin\theta)\tan\theta+7\cos\theta-29}}.$$

When $\theta=30°$, $v=59\sqrt{-3}$ inches, an impossible result.

$\therefore$ $GD>$than the intersection made by the particle on BD and indicates that the mud would never get 6 feet above the ground.

Let $\theta=60°$, $v=273.17$ inches$=22.76$ feet per second.

$t=5280\div22.76=231.98$ seconds$=3$ minutes, 51.98 seconds, time required to ride a mile.

101. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.

Find the center of gravity of a cone that has a specific gravity of 1 (one) at the top and 2 (two) at the base.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; WILLIAM W. LANDIS, A. M., Dickinson College, Carlisle, Pa.; H. C. WHITAKER, Ph. D., Manual Training School, Philadelphia, Pa.

Let $y=m(x-a)$ be the equation to the generator of the cone.

$$\text{Then }\ \bar{x}=\frac{\int\rho y^2xdx}{\int\rho y^2dx}=\frac{\int_a^{2a}\rho x(x-a)^2dx}{\int_a^{2a}\rho(x-a)^2dx}.$$

By the conditions of the problem, $\rho=x/a$.

$$\therefore\ \bar{x}=\frac{\int_a^{2a}x^2(x-a)^2dx}{\int_a^{2a}x(x-a)^2dx}=\frac{\frac{31a^5}{30}}{\frac{7a^4}{12}}=\frac{62a}{35}.$$

$\bar{y}=0$. $\dfrac{62a}{35}-a=\dfrac{27a}{35}=$the distance of the center of gravity from the vertex.